PHYSIOLOGICAL FLUID SYSTEMS MODELLING FOR NON-INVASIVE INVESTIGATION

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Summary The paper deals with the modelling and simulation of physiological fluid systems, especially the human vascular system. The described procedures are based on the electromechanical analogy making possible the application of electromagnetic theory. The physiological fluid segments are represented by the analogical equivalent electric transmission line sections. According to this analogy the electromagnetic propagation characteristics describe the mechanical properties of flowing fluids. The introduced modelling and simulation procedures enable to describe and visualize all states of dynamics of physiological processes, especially of the human haemodynamics. The obtained results serve as an important tool for non-invasive computer aided diagnostics.

Abstrakt Článok pojednáva o modelovaní a simulácii sústavy fyziologických tekutín, hlavne ľudského cievneho systému. Opísaný postup sa zakladá na elektromechanickej analógii umožňujúcej aplikáciu elektromagnetickej teórie. Segment fyziologickej tekutiny je reprezentovaný analogickým ekvivalentným úsekom elektrickej prenosovej linky. Podľa tejto analógie elektromagnetické prenosové charakteristiky opisujú mechanické vlastnosti prúdiacich tekutín. Uvedený postup modelovania a simulácie umožňuje opísať a zviditeľniť dynamické stavy fyziologických procesov, osobitne dynamiky ľudského krvného systému. Získané výsledky slúžia ako dôležitý nástroj pre neinvazívnu počítačovú diagnostiku.

1. INTRODUCTION

The most important physiological values of the human vascular system are the blood pressure, volume and flow, which can be represented by the electric potential, charge and current. The physiological system or its part can be represented by an electric equivalent model, created by a section of distributed parameters circuit. According to the transmission lines theory there have been used the differential equations for the mathematical interpretation of the electric model of the vascular system [1], [2]

$$-\frac{\partial P}{\partial z} = Z_l(\omega)\, I\,, \tag{1}$$

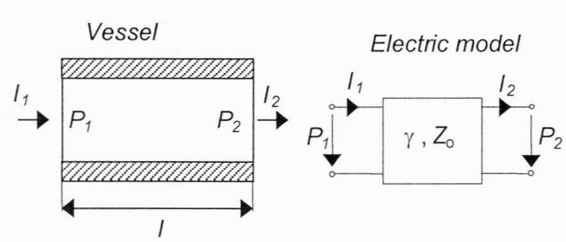

Fig. 1. The vessel segment and its electric model.

And $$-\frac{\partial I}{\partial z} = Y_t(\omega)\, P\,, \tag{2}$$

where P is the potential, I is the current, $Z_l(\omega)$ is the flow impedance in longitudinal direction and $Y_t(\omega)$ is the flow admittance in transversal direction.

The equivalent electric model, Fig. 1, could be expressed by the transmission parameters which are the wave (characteristic) impedance Z_o and the propagation factor γ in the form of input and output values according to the following equations

$$P_1 = P_2 \cosh \gamma z + Z_o I_2 \sinh \gamma z\,, \tag{3}$$

$$I_1 = (P_2 / Z_0) \sinh \gamma z + I_2 \cosh \gamma z\,. \tag{4}$$

Fig. 1. represents the short homogeneous vessel segment with its length l which complex impedance of the blood flow (in longitudinal direction) is given by the relation

$$Z_l'(\omega) = -\frac{j\,\omega\,\rho\,J_o(a\,r_o)}{\pi\, r_o^2\, J_2(a\,r_o)}\,, \tag{5}$$

where $Z_l'(\omega)$ is given per unit of length, ρ is the blood density, ω is the angular frequency, r_o is the radius of the vessel, J_o, J_2 are Bessel functions of the first kind.

In the case of the known dilatation of the vessel it is possible to calculate the wave propagation factor γ and the wave impedance Z_o from the relations

$$\gamma = \sqrt{Z_l'(\omega)\, Y_t'(\omega)} \quad \text{and} \quad Z_o = \sqrt{\frac{Z_l'(\omega)}{Y_t'(\omega)}}\,, \tag{6}$$

where $Y_t'(\omega)$ gives the elastic losses caused by the vessel cross section (in transversal direction).

In order to calculate longer homogeneous vessel segments we can involve them into the two-port circuit, the input and output quantities of which are given by the secondary transmission parameters of an electric line (γ and Z_o).

According to the expansion of Bessel functions J_o and J_2 the longitudinal impedance can be created by the cascade connection of resistances and inductances which are dependent on the vessel geometry, mainly on its radius r_o

$$\text{where } \lambda = -\frac{a\, r_o^2}{4} \text{ and } a = \sqrt{-\frac{j\,\omega\,\rho}{\eta}} \tag{8}$$

The expression (7) can be transformed to the chain fraction [3]

$$Z_l'(\omega) = \frac{8\,\eta}{\pi\, r_0^4}\left(1 + \frac{\lambda}{2.1} + \cfrac{\lambda/(3.2)}{1 + \cfrac{\lambda/(4.3)}{1 + \cfrac{\lambda/(5.4)}{1 + \cdots}}}\right),$$

which can be expressed in the form

$$Z_l'(\omega) = \frac{8\,\eta}{\pi\, r_0^4} + \frac{4\,\eta\,\lambda}{\pi\, r_0^4} + \frac{4\,\eta}{\pi\, r_0^4}\,\cfrac{1}{\cfrac{3}{\lambda} + \cfrac{1}{4 + \cfrac{1}{\cfrac{5}{\lambda} + \cfrac{1}{6 + \cdots}}}}.$$

This mathematical form can be interpreted as the impedance of the circuit at Fig. 2

$$Z_l'(\omega) = R_1' + j\omega L_1' + \cfrac{1}{\cfrac{1}{j\omega L_2'} + \cfrac{1}{R_2' + \cfrac{1}{\cfrac{1}{j\omega L_3'} + \cfrac{1}{R_3' + \ldots}}}}.$$

The cascade elements R_n' and L_n' will have values

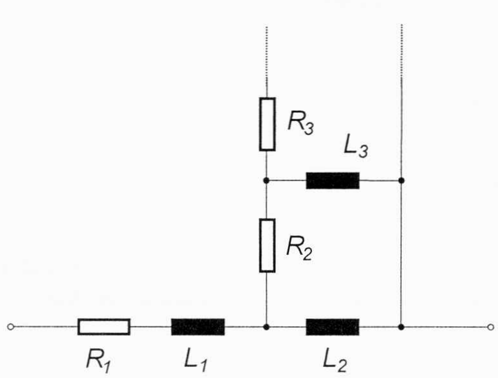

Fig. 2. The complex flow resistance equivalent

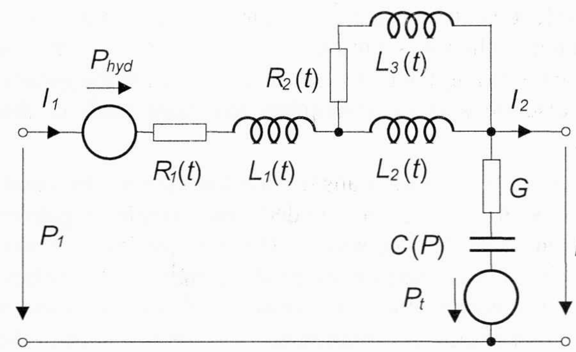

Fig. 3. The dynamic equivalent circuit of the vessel segment.

$$R_n' = \frac{8\,\eta}{\pi\, r_o^4}\, n \quad \text{and} \quad L_n' = \frac{\rho}{\pi\, r_o^2}\,\frac{1}{2n-1} \tag{9}$$

In the case of the stationary flow ($\omega = 0$), only the resistance R_1' will be present in the scheme.

By multiplication of the expression (9) by the segment length l we get the flow resistance of the segment.

The complex vascular tree consists of many various vessels both of homogeneous and non-homogeneous cross-section lines. For calculating reasons of the complex model the simulation of vessel tree branchings is performed by the adequate switching of the various vessel segments.

The system dynamic behaviour depends on the morphology and topology properties, but also on the non-linear characteristics due to the non-linear vessel walls resistance.

The dynamic equivalent representation of a vessel segment is shown in Fig. 3, where inductivities and resistances in the longitudinal direction represent the flow (hydraulic) resistance of the blood. The non-linear vessel wall properties are represented by the capacity, loss resistance and controlled source representing the hydrostatic pressure, which are placed in transversal branch.

The whole vessel tree has been replaced by the cascade connection of many vessel segments given by Fig. 3. The combinations of the time-discrete particular systems enable to determine the fluid mechanics in segments in the whole vascular system.

2. SYSTEM MATHEMATICAL REPRESENTATION

For the aim of the mathematical system description the modelling and following simulation have been performed in the state space. The non-linear processes have been described by means of both the continuity and the motion equations involving also the pressure-volume relations of vessel walls. The all mentioned influences result in usual differential equations and the corresponding equivalent electric image. According to the loop and node equations the time-continuous state space can be described. Using the equivalent transformation the general time-discrete system description has been used in this case.

On base of the electric transmission lines theory the vessel tree system has been divided into single segments represented by their equations. The input periodic or non-periodic impulse sequences produce transmitted pulses, which propagate along the vessel tree with its various inhomogeneities, *eg* branchings or stenosis and the multiple reflections determine the system dynamics. Using the state space methods the continuous distributed parameters systems are required to be transformed in the concentrated form. For the reason of an efficient numerical calculation the time continuous system representation was transformed into the time-discrete form.

The mathematical description of haemodynamics follows from the state space that enables very easy representation of the physical continuities and it offers suitable tool for the numerical analysis and simulation. But there exist limitations for its use, because only the systems with concentrated parameters can be calculated in the state space as the connection between the excitation and the response can be described by the usual differential and difference equations. The fluid mechanical systems are continuous ones with distributed parameters and they have to be characterized by partial differential equations. In general concentrated elements systems are represented by the following differential equations

$$\frac{d\,\bar{x}(t)}{d\,t} = \bar{f}(\bar{x},\bar{u},t)\ , \qquad \bar{y}(t) = \bar{g}(\bar{x},\bar{u},t)\ . \tag{9}$$

The state of such system is determined by the state values $x_i\,(t)$. It describes the influence of the input values $u_i\,(t)$ so that it is possible to calculate in every time the output values and the state values from the previous state and from the input values.

The properties of non-linear and time-dependent systems can be approximated piece-wise as linear and time-independent systems. The linear differential equations system in the basic form can be used

$$\frac{d\bar{x}(t)}{dt} = \mathbf{F}\bar{x}(t) + \mathbf{G}\bar{u}(t), \qquad \bar{y}(t) = \mathbf{C}\bar{x}(t) + \mathbf{D}\bar{u}(t), \tag{10}$$

where **F**, **G**, **C** and **D** are the system matrixes.

The transformation of the time-continuous systems into the time-discrete form can be expressed for the discrete time interval $t = k\,\Delta\,t$ by the following relations

$$\bar{x}(k+1) = \mathbf{A}\,\bar{x}(k) + \mathbf{B}\,\bar{u}(k), \qquad \bar{y}\,(k) = \mathbf{C}\,\bar{x}(k) + \mathbf{D}\,\bar{u}(k). \tag{11}$$

The calculation of the time-discrete system matrixes **A**, **B**, **C** and **D** from the time-continuous descriptions can be performed *eg* by the linear or equivalent transformations. The described method was used for the calculation of one vessel segment equivalent circuit according to the Fig. 3. The independent loops and nodes equations can be expressed by the time continuous system representation. The state values x_i are chosen in order to characterize the state of the linear and independent energy sources. The one capacity current and two inductivity voltages create the input values of the linear equations system, which describes entirely the dynamics of the analogous electrical circuit.

After determination of all matrixes elements we perform the transformation into the time-discrete system by means of the equivalent transformation. According to the equivalent transformation the time discrete matrixes **A** and **B** are given by the next equations

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \psi(\Delta\,t), \tag{12}$$

$$\mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \end{pmatrix} = \int_0^{\Delta t} \psi\,(t)\ \mathbf{G}\ d\,t\,, \tag{13}$$

where $\psi\,(t)$ is the transformation matrix.

For the single matrixes elements a_{ik} and b_{ik} are valid the following relations

$$a_{ik} = \sum_{j=1}^{3} k_{ik\,j}\ e^{\lambda_j \Delta t}\ , \tag{14}$$

$$b_{ik} = \sum_{n=1}^{3}\left(g_{nk}\ \sum_{j=1}^{3}\left(\frac{k_{inj}}{\lambda_j}\left(e^{\lambda_j \Delta t} - 1\right)\right)\right), \tag{15}$$

where λ_j are the eigen-values of the matrix **F** and k_{ikj} are the values resulting from the relation for the elements of the transformation matrix

$$\psi_{ik}\,(t) = \sum_{j=1}^{3} k_{ik\,j}\ e^{\lambda_j \Delta t}\ .$$

In this way the equivalent time-discrete system description of the single vessel segment was calculated. Following from the linear basic systems the non-linear system properties of vessel segments will be transformed and involved into the described model as non-linear parts of systems. Block representation of the realization of one vessel segment in time-discrete area is given by Fig. 4. [5], [6].

The system matrixes are dependent on the values of the equivalent circuit elements R_1, R_2, L_1, L_2, L_3, G and C. In general the elements values are the functions of the time but within one time interval Δt can be considered as constants. The time-discrete system matrixes change their values in dependence on k, but they are taken as constants within one time interval. The non-linear properties are represented by $f(x_1)$ unit [4].

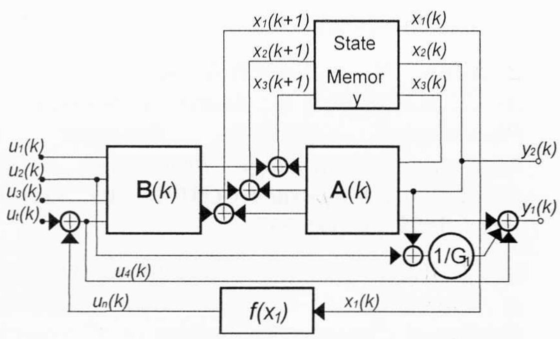

Fig. 4. Block representation of the time-discrete realization of the vessel segment.

The described method for the modelling of single vascular segments enables to create the modelling methods of the vascular network space structures. The model description of the network topology was obtained by means of angiographic representation. The base of the realization of the stochastic topology model consists in the growth of an initial tree of a real vessel tree. The growth algorithms describe the growth within the determined volume under hypothetical optimization criteria. The aim of the modelling is to keep both the real input behaviour

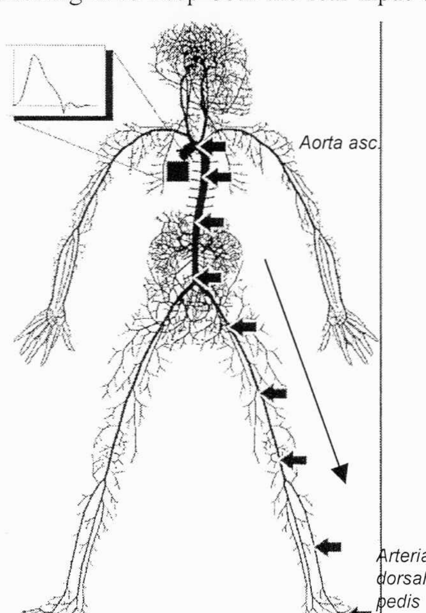

Fig. 5. Virtual morphological model of the human circulatory system.

and the division of the pulsating blood flow in various perfusion areas [2]. The backward influences of control processes on the blood flow dynamics in the big vessels have been simulated and the local blood division up to the capillary level can be also represented in this way.

3. SIMULATION RESULTS

The entire tree of the morphological model of the human circulatory system, Fig. 5., consist of 4105 segments and it is divided into sub-trees of organs according to the functional aspects.

The signal source simulates the heart output under usual conditions (75 beats per minute, heart mean volume HMV= 5800ml/min) [4]. The typical simulation results of blood pressure and blood flow characteristics in selected vascular levels are shown in Fig. 6 and 7. Fig. 6 shows the pressure-pulse propagation from the heart to the

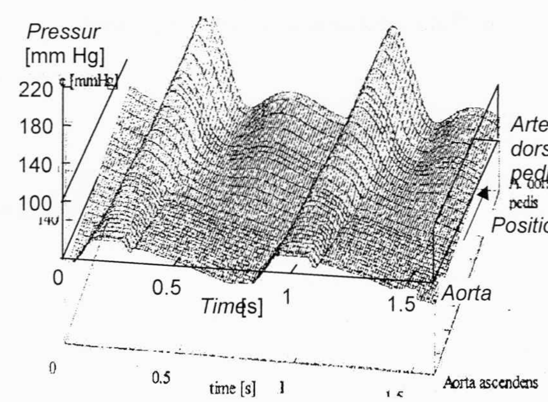

Fig. 6. Pressure-pulse wave propagation under usual conditions.

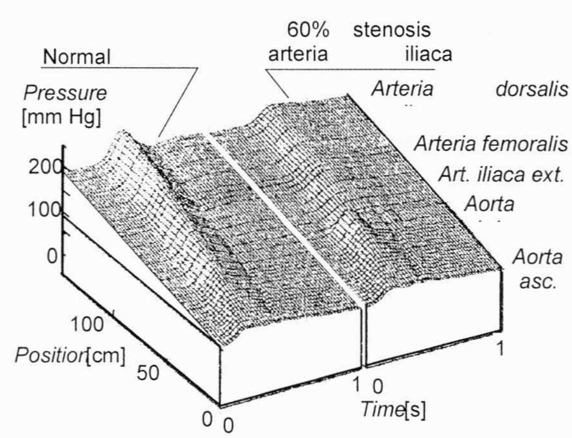

Fig. 7. Pressure-pulse wave propagation with 60% stenosis.

foot, Fig.7 shows the same dependence but with a simulated 60% stenosis in the Arteria illiaca externa. Both results were obtained for a standing position, so that hydrostatic pressure component is added, leading to an increase in the arteria elasticity module in the lower extremities. The pulse propagation in vessel tree with stenosis shows the pathophysiological deformation of the pressure pulse wave.

4. CONCLUSION

The modelling and simulation method of dynamic physiological fluid systems using electromagnetic theory of transmission lines have been introduced and performed in the paper. According to the described propagation properties both the electric and the mathematical models of the vascular system were created. The successful imitation of blood pressure and blood flow characteristics leads to the conclusion that the simulation procedure is theoretically able to describe all states of human haemodynamics adequately. Therefore the simulation method in combination with the experiments (e.g. angiological measurements) represents the computer aided modelling technique for non-invasive investigations of fluid physiological systems, or for non-destructive evaluation of fluid mechanical systems in general.

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